Classical origins of stabilization in circularly polarized laser fields

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We investigate the interaction of a two-dimensional model atom with an intense, high frequency circularly polarized laser pulse. As the laser intensity is increased, the ionization rate initially increases, then decreases dramatically, with the electron wavefunction developing an asymmetric ring form which rotates with the electric field. We provide evidence that this waveform is due to localization of the electron onto nonlinear classical structures.

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The advent of lasers producing electric fields on the order of interatomic electric fields has led to the discovery of many unexpected nonlinear phenomena [1]. One of these effects is the stabilization of atoms in intense, high-frequency laser fields, characterized by a *decrease* in ionization probability as the laser intensity is *increased.* The field and frequency regime where stabilization is known to occur, for the well studied case of linear polarization, is approximately given by $F^2/2\omega^2 \gtrsim \omega \gtrsim |E_o|$, where F and ω are the laser field and frequency in atomic units ($\hbar = e = m = 1$), and $|E_o|$ is the ground state (g.s.) ionization energy. This phenomena has only been observed through careful preparation of initial atomic states [2–5], as observation of stabilization using actual ground states, such as hydrogen ($|E_o| = 13.7eV$), would require an intense, high energy photon source. Although stabilization was originally predicted using time independent Floquet methods [6], it also can also occur for atoms subject to intense, short laser pulses [7,8]. Using high frequency Floquet theory, Pont, Gavrila, and co-workers [9] presented the first computations indicating stabilization in laser driven hydrogen. For linear polarization, they found the wavepacket develops a dichotomous form, a characteristic later confirmed by time dependent studies [10]. Pont and Gavrila also found stabilization for hydrogen driven by a circularly polarized laser [11], with expected toroidal wavefunctions [12]. Recent time dependent investigations have found these waveforms, and revealed new behaviors [13,14].

Classical effects underly many nonlinear quantum phenomena. For the case of stabilization in a linearly polarized laser field, the classical basis for the form of the wavefunction may be understood from the one dimensional "soft-core" Rydberg model atom [15]. This one dimensional atom and field form a $1\frac{1}{2}$ degree of freedom system whose phase space may be visualized with a simple strobe plot [16]. As the laser field is increased into the stabilization regime, the resonance in the neighborhood of the core undergoes a bifurcation and forms two resonance islands separated by a hyperbolic fixed point and chaos. In particular, localization to the resonance islands produces the characteristic long-lived dichotomous form. By transforming to the frame of an electron moving in response to the laser field alone, known as the Kramers-Henneberger (KH) frame, one may isolate the dynamics which are due to the atomic core. In this frame, a stabilized electron will drift back and forth between turning points located a distance $2F/\omega^2$ apart, being passed many times by the rapidly oscillating core. However, impulses due to the passing core are small, and act to stabilize the particle [17]. Indeed, Jensen and Sundaram, using the fact that these impulses are approximately periodic for large F/ω^2, were able to derive a map used which describes the stabilization dynamics well [18].

For stabilization in circularly polarized fields, we also expect a classical basis for quantum waveforms and lifetimes. In this paper, we investigate characteristic features of stabilized waveforms in the case of circular polarization, using time dependent methods. Then, by studying the corresponding classical model, we establish the classical origin of these features, also indicating how classical dynamics affects the population of these quantum structures.

Quantum dynamics-In order to determine the possible forms for the stabilized wavefunctions in circular polarization, we take the typical "wavepacket dynamics" approach [10], using the dipole approximation with classical radiation field. We adopt a two dimensional version of the soft-core Rydberg model used by Patel, *et al* [13]. By restricting to the plane of polarization and eliminating the Coulomb singularity, this model allows integration of the time dependent Schrödinger equation in two dimensions without basis expansions. Stabilized wavepacket lifetimes predicted using this two dimensional model are in good agreement with lifetimes attained via fully three dimensional time dependent and time independent methods [19,10,14]. The electron Hamiltonian is

$$H = \frac{1}{2}\left(\hat{p}_x^2 + \hat{p}_y^2\right) - \frac{1}{\sqrt{\alpha^2 + x^2 + y^2}} + g(t)F(\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)), \qquad (1)$$

where α is a smoothing parameter, $\hat{p}_x$ and $\hat{p}_y$ are momentum operators, $\hat{x}$ and $\hat{y}$ are position operators, and $g(t)$ describes the laser field turn-on. We choose $\alpha = 0.8$, so that the ionization potential is the same as that of hydrogen [13], and set the laser frequency $\omega = 1.2$. The Schrödinger equation is integrated using Crank-Nicholson finite differencing and absorbing boundary conditions [14]. For determination of the lifetimes, we start with the g.s. wavefunction and turn on the laser field with $g(t) = \sin^2(\frac{\omega t}{4n})$, over the time interval $0 < t < 2\pi n/\omega$. After time $t = 2\pi n/\omega$, we set $g(t) = 1$. As the interaction is turned on, a portion of the wavefunction leaves the grid, becoming ionized. However, when stabilization occurs, some probability becomes localized for long times, its amplitude exhibiting a well defined exponential decay. In Fig.1, we show the increase in lifetime with field strength, for both linear and circular laser polarization. The circular case shows a dramatic increase in lifetime as the field is increased above $F \approx 3a.u.$ ($I \approx 3 \times 10^{17} W/cm^2$), indicating stabilization is much stronger in this regime than that expected for linearly polarized fields [10]. Note that Ref. [19] reports lifetimes for circular polarization, but only for fields $F \leq 2.4a.u.$

In Fig.2 we show a snapshot of a stabilized waveform for circular laser polarization. The field strength is $F = 3.75$ and the time is $t = 85.5$, approximately 16.3 laser cycles. We find a ring of enhanced probability, with radius of approximately F/ω^2. Probability on the ring is asymmetrically distributed with maximum and minimum values occuring on opposite sides of the ring. The ring is displaced along the field a distance $\approx F/\omega^2$, and rotates, locked in phase with the field, about the origin. To illustrate this, in Fig.3 we show the time evolution of the probability density over one field cycle along the $y = 0$ axis. At time $t = 47.1$ (9 laser cycles), the probability is peaked just inside $x = 2F/\omega^2$. This maximum then rotates away from the $y = 0$ plane, reappearing near $x = -2F/\omega^2$, one half cycle later. The maximum probability disappears and reappears again along the x axis at $x \approx 2F/\omega^2$, completing the cycle. This rotating, stabilized waveform remains intact until all probability decays out of the grid. Since the probability maximum in the rotating ring occurs near $2F/\omega^2$, this peak has an excursion of almost $4F/\omega^2$, twice the expected value.

Now we consider the waveform behavior seen by Patel, *et al.* In fig. 1(c) of ref. [13], they show a snapshot which consists of a ring of enhanced probability amplitude, surrounding a higher probability central peak. The radius of the outer ring is $F/\omega^2 = 15.0/1.0^2 = 15.0$. The central peak follows a circular orbit of radius F/ω^2 about the origin, in phase with the high frequency field. The ring structure, which is centered on this peak, also rotates about the origin, with one edge intersecting the core. This ring structure agrees with the expected toroidal waveform from the time averaged KH ground state [13]. The "central" probability peaks, located at the origin of the KH frame, are due to superpositions of Floquet states excited during the turn-on.

Classical dynamics-The time dependent classical Hamiltonian corresponding to Eqn.1 appears to have $2\frac{1}{2}$ degrees of freedom. However, there is an additional constant of the motion which reduces the dynamics to a space with 2 degrees of freedom. This allows construction of a Poincaré surface of section [16]. Adopting polar coordinates, and making a canonical transformation: $(p_\rho, \rho, p_\theta, \theta, t) \longrightarrow (P, \rho, J, \phi)$, via the generating function $F = J(\theta - \omega t)$, we obtain the time independent Hamiltonian

$$H = \frac{P^2}{2} + \frac{J^2}{2\rho^2} - J\omega - \frac{1}{\sqrt{\alpha^2 + \rho^2}} + F\rho\cos\phi = \Omega. \quad (2)$$

The dynamics of this system are quite different from the dynamics of the $1\frac{1}{2}$ degree of freedom system corresponding to linear polarization. We again set $\alpha = 0.8$, which retains features of the classical dynamics having phase space structures larger than $\hbar$, and therefore can be seen by the quantum system. For a given value of the "quasienergy" Ω, we plot the radial momentum P and position ρ each time the angle variable ϕ reaches zero, mod π. If a conserved quantity exists in addition to the quasienergy, the points in this surface will trace out one dimensional curves, otherwise, the points will scatter about the quasienergy surface. The angular momentum J is not conserved. Therefore, the repulsion at the core will vary in time. We take initial conditions corresponding to orbits which will pass near the core with zero drift velocity. These are $P_o = 0$, $\phi_o = 0$, $\rho_o \approx 2F/\omega^2$, and $\Omega \approx F^2/2\omega^2$, with the value J_o fixed by the quasienergy. In Fig.4, we show the classical surface of section for $F = 3.75$, the field strength used to obtain our quantum results. For quasienergy $\Omega = 3.125$ (upper box), there are regular structures and very slowly ionizing regions near $x = 0$ and $x = 2F/\omega^2$, the location of the rotating ring waveform found in the quantum case. Also, at this quasienergy, there is an excluded region in phase space at the center of this structure. For quasienergy $\Omega = 5.0$ (lower box), the location of the stabilization region remains as before, however, the underlying structure has become much more chaotic and the area where classical accumulation may occur has increased. This enhanced region of "sticky motion" is not seen in the classical structures underlying stabilization in linear polarizations [18].

The underlying classical structure is also present for the ring structure reported by Patel, *et al.* We show this in Fig. 5. In the upper box, we plot the surface of section for $F = 15.0$, $\omega = 1.0$, and $\Omega = 110.0$. The phase space is dominated by resonance structures located near the core, and near twice the classical excursion, with an excluded region between them. At a slightly higher quasienergy, as shown in the lower box for $\Omega = F^2/2\omega^2 = 112.5$, we can see an unstable fixed point at $x \approx F/\omega^2 = 15.0$. The central peak in Patel's waveform is localized on this unstable fixed point. Thus, we find the origin of stabilized ring waveforms in circular laser polarization are classical nonlinear structures created by the presence of the core [16].

Population of these stabilized structures can also occur classically. Starting with an ensemble of initial points localized near the origin with *physical* energy $E = -0.5$, we turn on the laser field exactly as in our quantum case. At the end of the turnon, we found most quasienergies had been swept into the range $\Omega = 4.0 \pm 1.0$, corresponding to values associated with our stabilized waveform. The particles with small initial momenta typically accumulate in the stable structures. In the case of Patel's parameters, we find that an initial ensemble is swept into the quasienergy interval $\Omega \approx 111.0 \pm 3.0$, again matching the values at which the stabilized structure is present.

In conclusion, we have found that stabilization in circularly polarized laser fields is accompanied by the formation of ring waveforms of radius F/ω^2, displaced a distance F/ω^2 along the laser field. These structures are locked in phase with the laser field, similar to the "Trojan wavepacket" states seen in strong circularly polarized microwave fields [20]. However, for the fields and frequencies we consider, the underlying classical structure has undergone a bifurcation [16]. Localization of the wavefunction onto this classical structure produces the characteristic ring waveforms. We also have shown that these ring waveforms may be asymmetrically populated, leading to enhanced wavepacket excursion, and that population of these new stabilized structures may occur classically.

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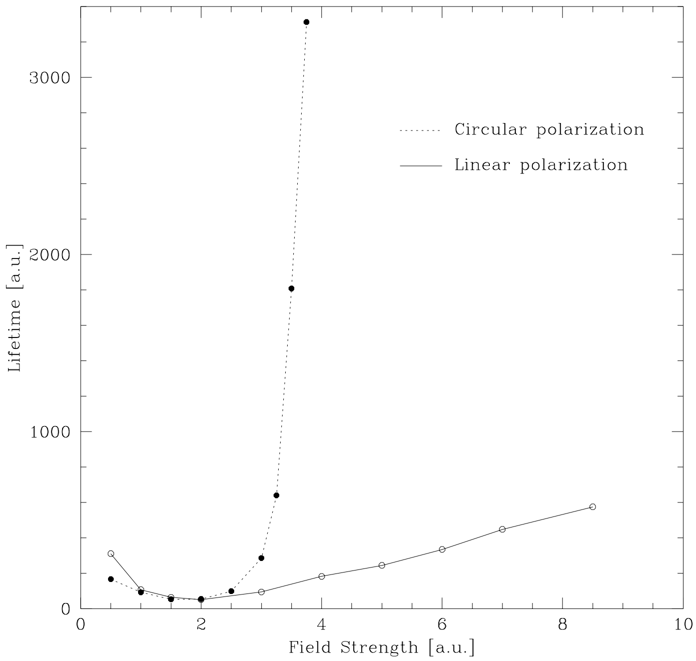

FIG. 1. Stabilized state lifetime, as a function of laser field strength, for the two dimensional model atom. The laser frequency is $\omega = 1.2$. All quantities are in atomic units. We see dramatic lifetime enhancement for circular polarization.

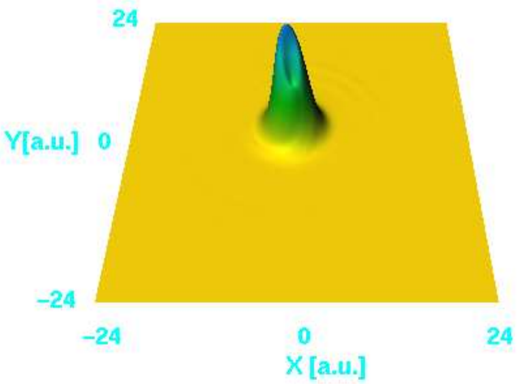

FIG. 2. Snapshot of the stabilized waveform in the laboratory frame for circular polarization. The laser field strength is $F = 3.75$, and the time is $t = 85.5 a.u.$, about 16.3 laser cycles. We see a ring of probability, with oppositely placed minimum and maximum values. The maximum occurs at $\rho \approx 2F/\omega^2$, and the minimum occurs near the origin.

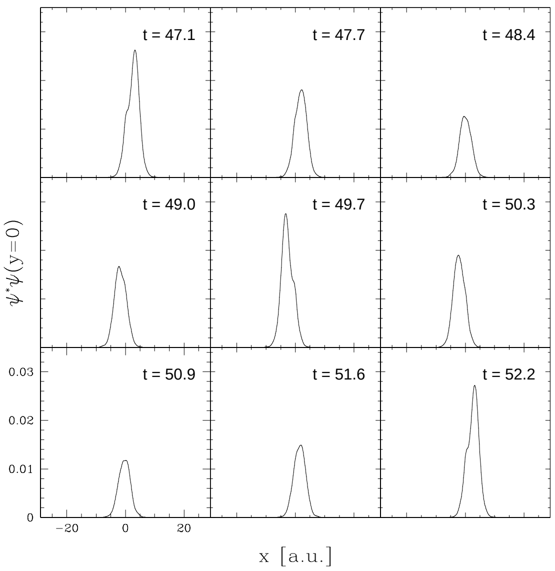

FIG. 3. Time development of the stabilized electronic wavepacket shown in Fig.2, over one field cycle, along the slice $y = 0$. The peak probability starts just inside $x = 2F/\omega^2$, disappears, reappears at $x \approx -2F/\omega^2$, disappears again, and returns near $x = 2F/\omega^2$. Thus, this structure rotates in phase with the high frequency laser field, with peak probability displaced a distance $2F/\omega^2$ opposite the field direction.

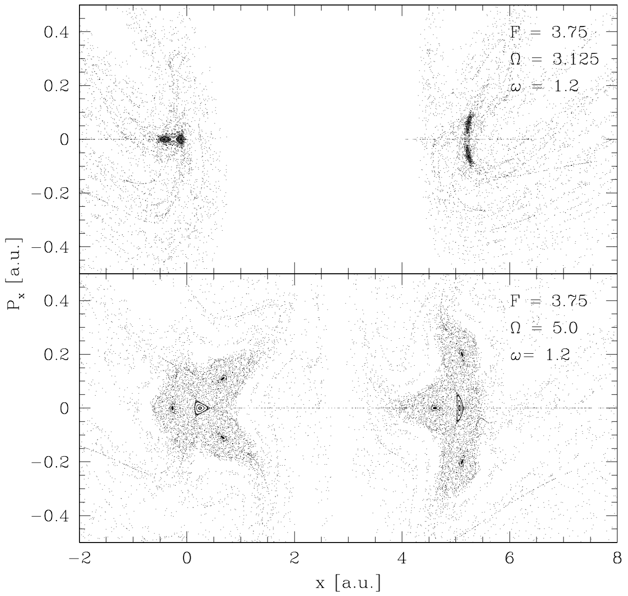

FIG. 4. Classical stabilization structure for circular laser polarization. Upper box: Poincaré surface of section taken along the rotating laser field for field $F = 3.75$ and quasienergy $\Omega = 3.125$. We find regular structures and chaotic accumulation regions near $x = 0$ and $x = 2F/\omega^2$. Lower box: Poincaré surface of section for $F = 3.75$ and $\Omega = 5.0$. The accumulation region has become increasingly chaotic via the process of resonance overlap.

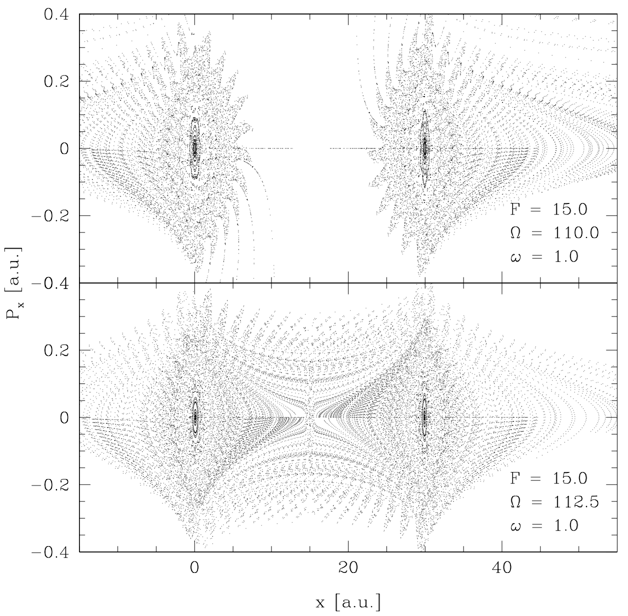

FIG. 5. Classical stabilization structure corresponding to the quantum ring waveform seen by Patel, et al. Upper box: Poincaré surface of section along the laser field for $F = 15.0$, $\omega = 1.0$, and quasienergy $\Omega = 110.0$. Again, there are regular structures and chaotic accumulation regions near $x = 0$ and $x = 2F/\omega^2 = 30.0$. Lower box: Poincaré surface of section as above, for $\Omega = 112.5$. The fixed point near $x = F/\omega^2 = 15.0$ is unstable.